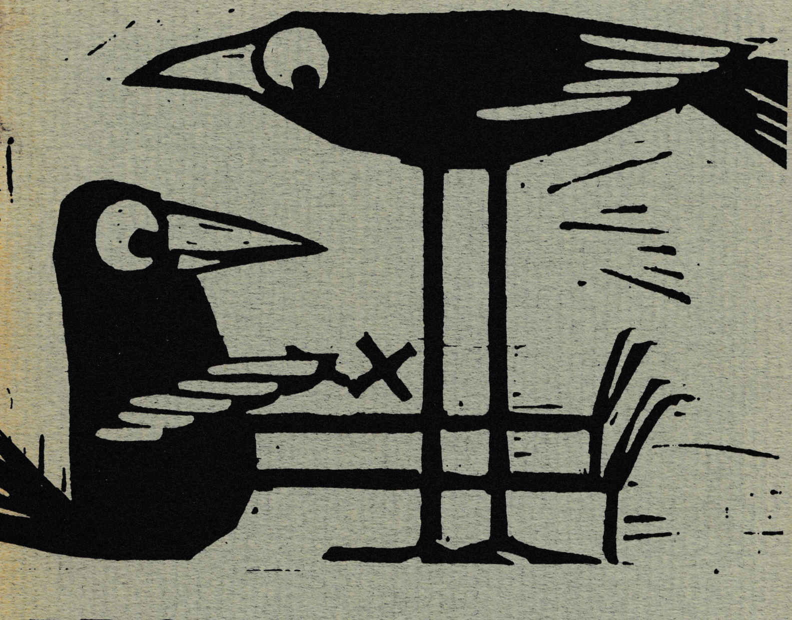


PROBLEMATICAL RECREATIONS 5

PROBLEMATICAL RECREATIONS^5

Fifth in a series

⁂

The forty-one posers that follow were, for the most part, thoughtfully contributed by our readers. We pass them along for your mathematical entertainment.

Some require simple reasoning while others might challenge a professional mathematician. In all the emphasis has been on conciseness of statement, elegance of solution, imaginative appeal.

When you have arrived at an answer, check with ours in the back of the booklet. May we hope they agree.

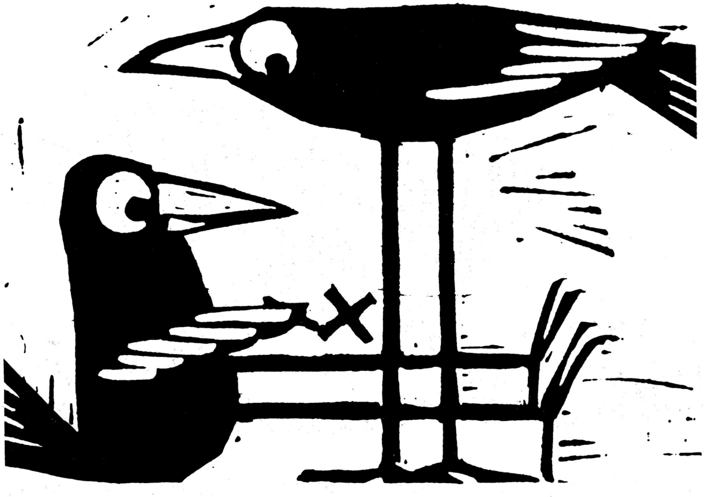
⁂

Courtesy of Litton Industries, Beverly Hills, California

1.

A man leaves from the point where the prime meridian crosses the equator and moves forty-five degrees northeast by geographic compass which always points toward the north geographic pole. He constantly corrects his route. Assuming that he walks with equal facility on land and sea, where does he end up and how far will he have travelled when he gets there? *— Contributed*

2.

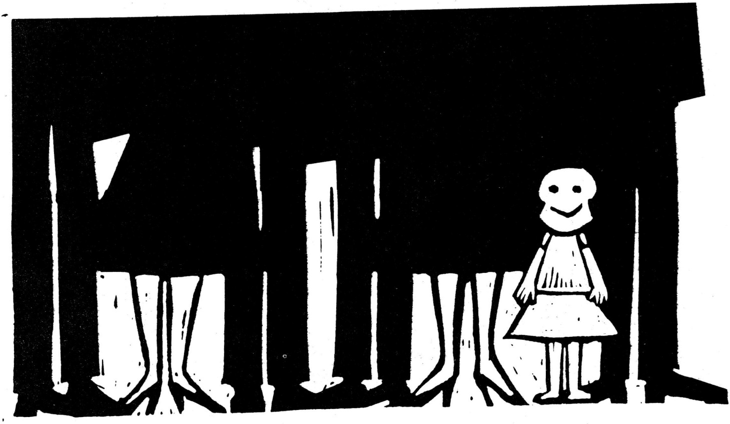


The game of reverse tic-tac-toe (known to some as toe-tac-tic) has the same rules as the standard game with one exception. The first player with three markers in a row loses. Can the player with the first move avoid being beaten? —*Contributed*.

3.

Johann Jungfrau, the famous mountain climber, was traveling through the Trondheim timber country one day. Quite by accident he dropped his trusty Alpenstock, an unusually straight stick, near the buzz saws where, in two shakes of a yak's tail, it was neatly cut into three pieces. What is the probability that these three pieces can be placed together to form a triangle? *Contributed*

4.



The family of a Quality Control Engineer consisted of 1 grandmother, 1 grandfather, 2 fathers, 2 mothers, 4 children, 3 grandchildren, 1 brother, 2 sisters, 2 sons, 2 daughters, 1 father-in-law, 1 mother-in-law, and 1 daughter-in-law. What is the smallest possible number of persons in his family? —*Contributed*

5.

If a certain six digit number is split into two parts, one constituting the first three digits and the other the last three digits, and the two parts are added and the resulting sum squared, it is found that the product is the original six digit number. What is the number?

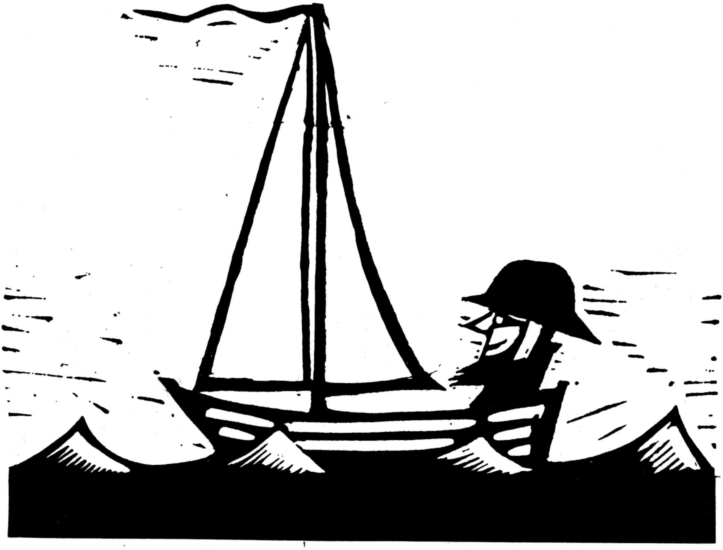
—Contributed

6.

My house is on a road where the numbers run 1, 2, 3, 4 . . . consecutively. My number is a three digit one and, by a curious coincidence, the sum of all house numbers less than mine is the same as the sum of all house numbers greater than mine. What is my number and how many houses are there on my road?

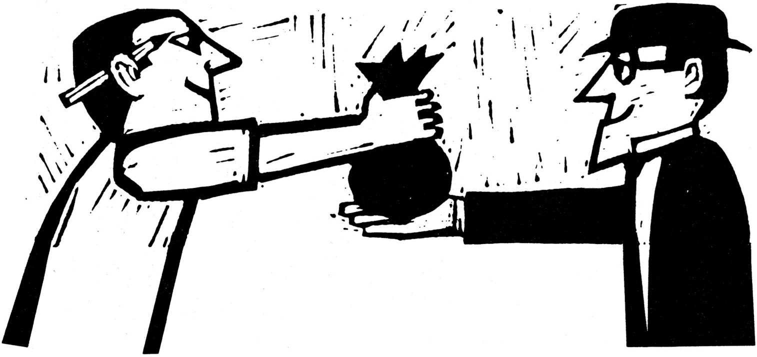
– *Contributed*

7.



At the local yachting club they were checking over their equipment in preparation for the coming season. Woodson, a mathematically minded member, remarked that the two sails on his boat, though different in area, shared a curious property: the area of each in yards was equal to its perimeter in yards. Both sails, it should be mentioned, were of the usual right triangular shape with integral sides. What must have been their areas?—*Contributed*

8.



Dr. Furbisher LaRouche, the noted mathematician, was shopping at a hardware store and asked the price of certain articles. The salesman replied, "One would cost 10 cents, eight would cost 10 cents, seventeen would cost 20 cents, one hundred and four would cost 30 cents, seven hundred and fifty six would also cost 30 cents, and one thousand and seventy two would cost 40 cents." What was Dr. LaRouche buying? —*Contributed*

9.

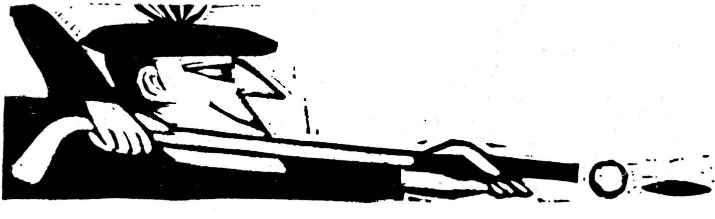
Near the town of Lunch, Nebraska there is a large triangular plot of land bounded by three straight roads which are 855, 870, and 975 yards long respectively. The owner of the land, a friend of mine, told me that he had decided to sell half the plot to a neighbor, but that the buyer had stipulated that the seller of the land should erect the fence which was to be a straight one. The cost of fences being high, my friend naturally wanted the fence to be as short as possible. What is the minimum length the fence can be?

—*Contributed*

10.

Find 1000 consecutive nonprime numbers. —*Contributed*

11.



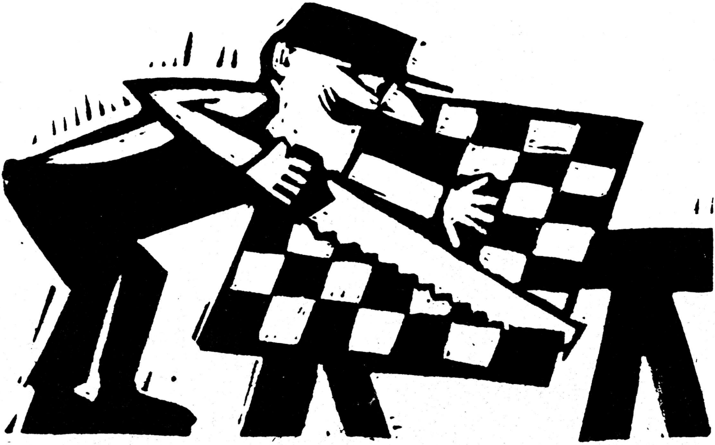
An elimination golf tournament is held for (n + 1) players. How many golf matches will transpire?

—Contributed

12.

Find the smallest integer which is such that if the digit on the extreme left is transferred to the extreme right, the new number is three and a half times the original number. —*Contributed*

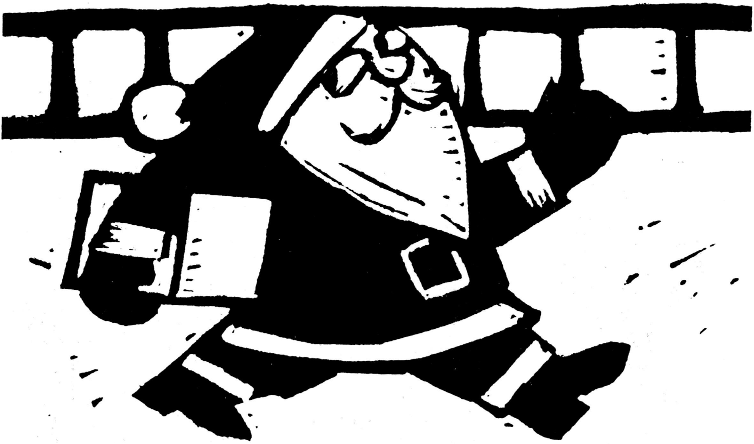
13.



Two squares are removed from opposite corners of a checkerboard leaving 62 squares. Can the checkerboard be filled with 31 dominoes, each domino covering two adjacent squares?

—*Contributed*

14.



A cubic box with sides ‘a’ feet long is placed flat against a wall. A ladder ‘p’ feet long is placed in such a way that it touches the wall as well as the free horizontal edge of the box. If $a = 1$ and $p = \sqrt{15}$, calculate at what height the ladder touches the wall, using quadratics only. *—Contributed*

15.

Supply the missing number in the following sequence: 10, 11, 12, 13, 14, 15, 16, 17, 20, 22, 24, ——, 100, 121, 10,000.

— Contributed

16.

Between Kroflite and Beeline are five other towns. The seven towns are an integral number of miles from each other along a straight road. The towns are so spaced that if one knows the number of miles a person has traveled between any two towns he can determine the particular towns uniquely. What is the minimum distance between Kroflite and Beeline to make this possible?

– Contributed

17.

How many nine digit numbers are divisible by 11, no digit equal to zero and no two digits alike? *– Contributed*

18.

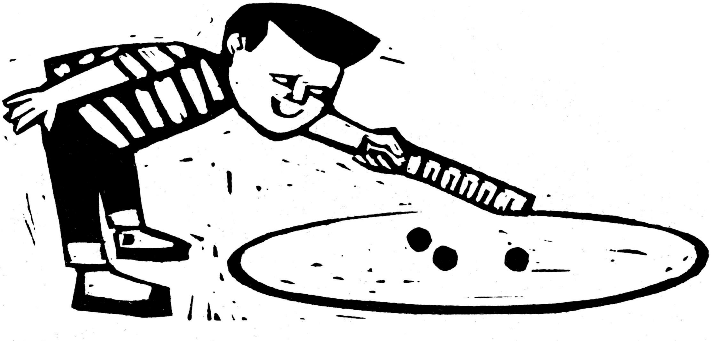
Noel Wentworth-Longmore, the famous Oxford rower, was rowing upstream one Sunday when his favorite rowing cap fell in the water. So absorbed was he in one of Housman's poems that it was ten minutes before he discovered his cap missing. He turned around and recovered the cap one mile downstream from where he initially lost it. Assuming constant speed, and no allowance for turn around, how fast was the river flowing? —***Contributed***

19.

Dr. Irving Weiman, who is always in a hurry, walks up an up-going escalator at the rate of one step per second. Twenty steps bring him to the top. Next day he goes up at two steps per second, reaching the top in 32 steps. How many steps are there in the escalator?

– *Contributed*

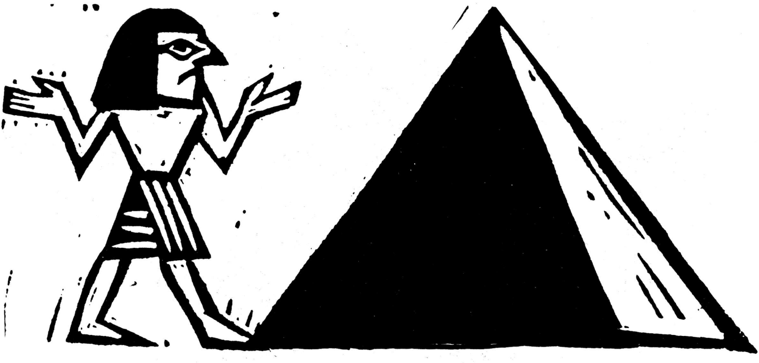
20.



From a point P outside a given circle, construct, using a straightedge only, the perpendicular to a given diameter of the circle.

—Contributed

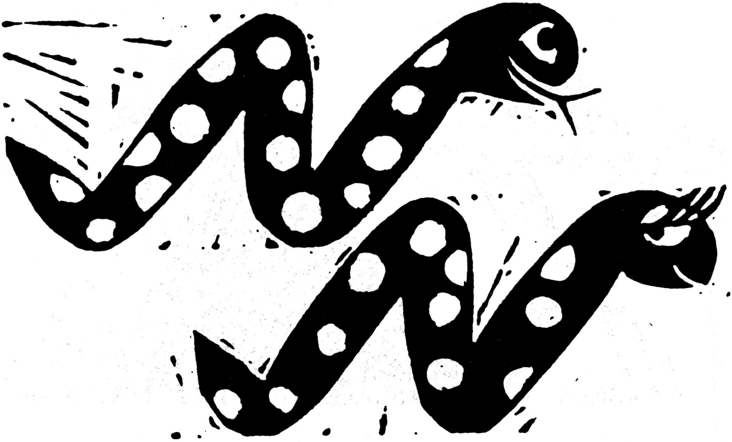
21.



Two similar triangles with integral sides have two of their sides the same. The third sides differ by 387. What are the lengths of the sides?

—Contributed

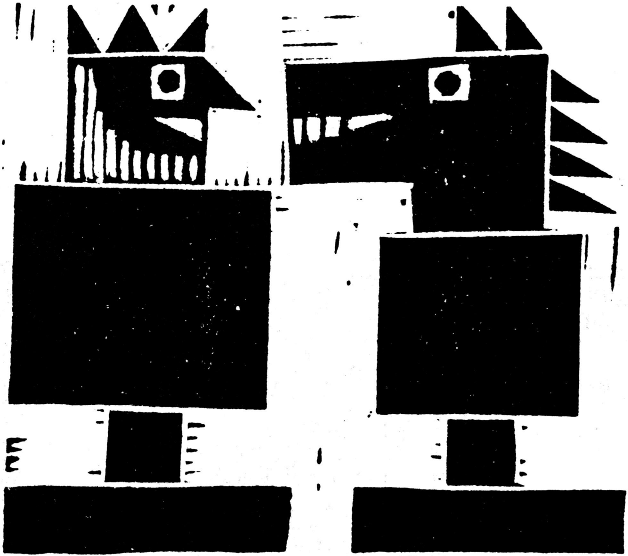
22.



The integers 1, 3, 8, N have the property that the product of any two when added to unity yields a square. What is N?

— *Contributed*

23.

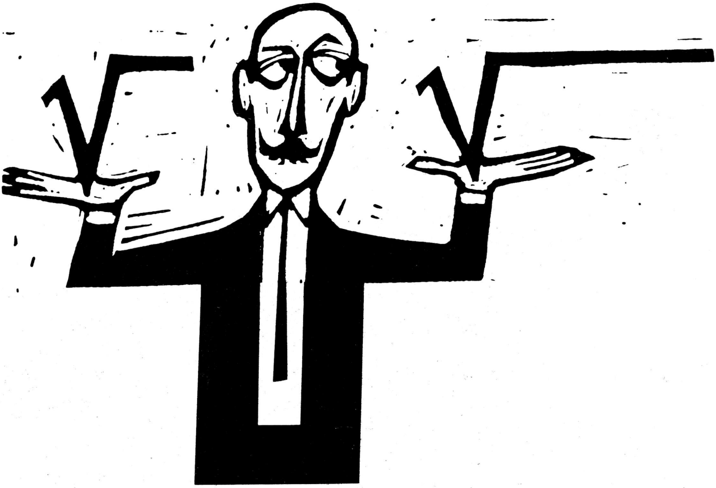


A modernistic chess set has pieces in various geometrical shapes. In particular, both the KING and the KNIGHT are squares of integers. What numbers could these represent if each letter is replaced by a different digit? *—Contributed*

24.

In Byzantine basketball there are 35 scores which are impossible for a team to total, one of them being 58. Naturally a free throw is worth fewer points than a field goal. What is the point value of each? *—Contributed*

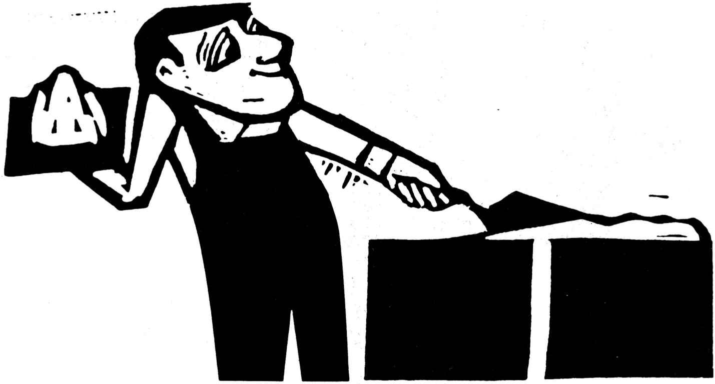
25.



Which is larger, the tenth root of ten or the cube root of two?

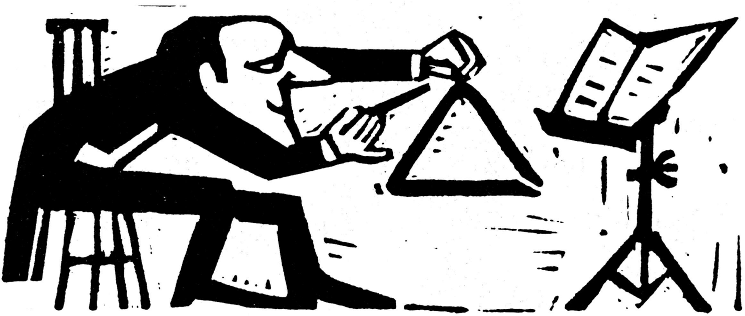
—Contributed

26.



Two cubes with integral sides have their combined volume equal to the combined length of their edges. What are the dimensions of the cubes? —*Contributed*

27.



A scalene triangle ABC which is not a right triangle has sides which are integers. If $\sin A = \frac{5}{13}$, find the smallest values for its sides, i.e., those values which make the perimeter a minimum.

– Contributed

28.

The planet Octerra is divided into eight countries, each occupying an octant, (thus each country borders three others). In how many ways can a traveler visit each of the other countries once and only once, returning to his home country only at the end of his trip?

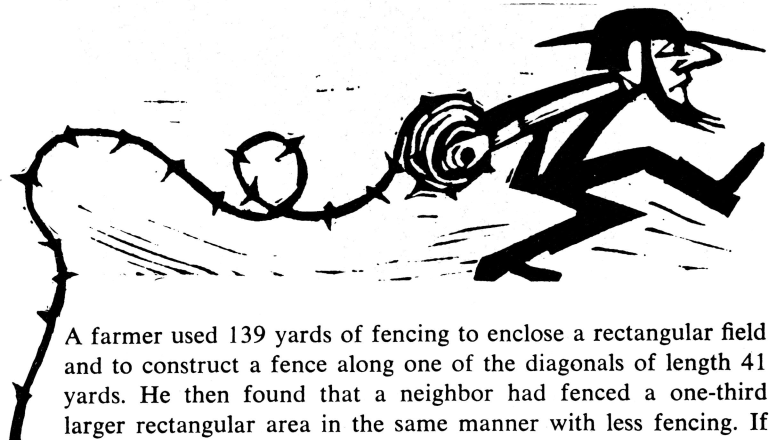
—Contributed

29.

Gherkin Gesundheit, a brilliant graduate mathematics student, was working on an assignment but, being a bit absent-minded, he forgot whether he was to add or to multiply the three different integers on his paper. He decided to do it both ways and, much to his surprise, the answer was the same. What were the three different integers?

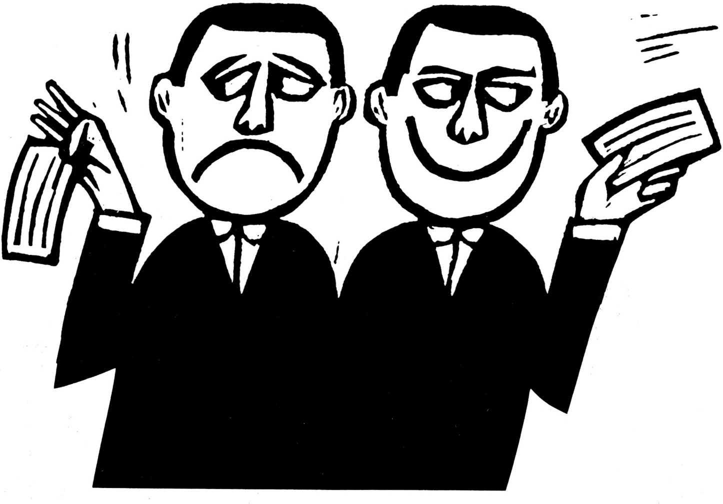
– Contributed

30.



A farmer used 139 yards of fencing to enclose a rectangular field and to construct a fence along one of the diagonals of length 41 yards. He then found that a neighbor had fenced a one-third larger rectangular area in the same manner with less fencing. If all dimensions are integral yards, what are the dimensions of the neighbor's field? *—Contributed*

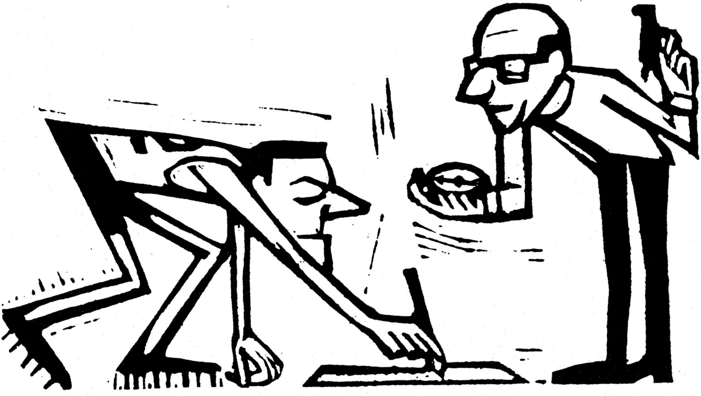
31.



Fourbisher and LaRouche started working for different firms at the same salary. Last year Fourbisher had a raise of 10%, and LaRouche had a drop in pay of 10%. This year Fourbisher had the 10% drop and LaRouche the 10% raise. Who is making more now?

—Contributed

32.



You have just thirty seconds to write down six different six digit numbers, each of which is divisible by 7, 11, and 13. Done it? Splendid! What's the secret? *—Contributed*

33.

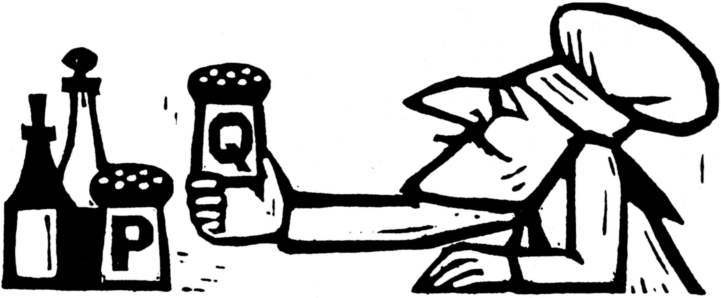
A man has red, gray and black flagstones for making a walk. He wants no two consecutive stones to be the same color, no consecutive pair of stones to have the same two colors in the same order, no repetition of three consecutive colors, etc. He starts out laying first a red stone, then a gray, and continues until he finishes laying the seventh stone. He then finds himself stymied and unable to use any stone for the eighth without repetition of some color pattern. What were the colors of the first seven stones?

— *Contributed*

34.

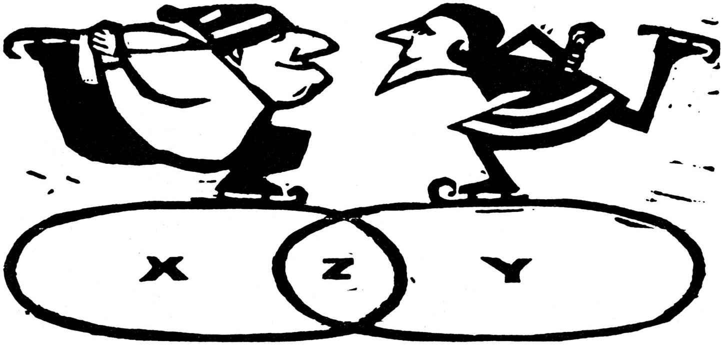
What letter follows OTTFFSSE___? *– Contributed*

35.



The sum and difference of two squares may be primes: $4 - 1 = 3$ and $4 + 1 = 5$; $9 - 4 = 5$ and $9 + 4 = 13$, etc. Can the sum and difference of two primes be squares? If so, for how many different primes is this possible?
—Contributed

36.



In a certain community there are 1000 married couples. Two-thirds of the husbands who are taller than their wives are also heavier and three-quarters of the husbands who are heavier than their wives are also taller. If there are 120 wives who are taller and heavier than their husbands, how many husbands are taller and heavier than their wives? *– Contributed*

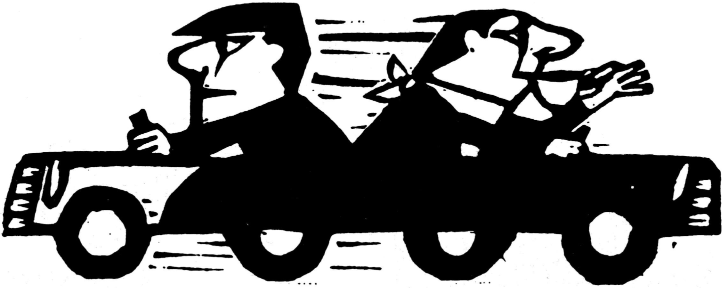
37.

For what values of n is $(11 \times 14^n) + 1$ prime? *– Contributed*

38.

The area and volume of a certain sphere are both 4-digit numbers times π. What is the radius of the sphere? – *Contributed*

39.



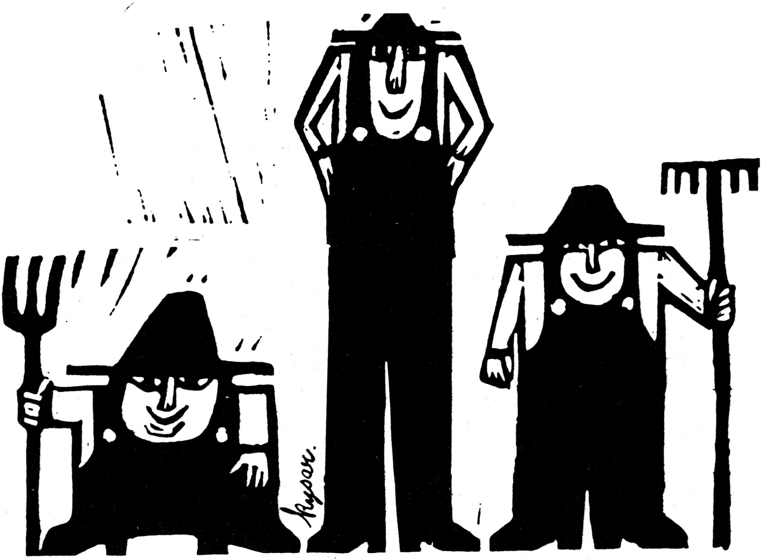
Furbisher lives in Canoga Acres and works in Beverly Flats. LaRouche lives in Beverly Flats and works in Canoga Acres. They usually leave their respective homes at the same time, and pass each other at Sam's Hamburger Shack. (Furbisher drives twice as fast as LaRouche) On a day when Furbisher's wife Formica spoiled the eggs and he left 5 minutes late, they passed each other at the gas station, two miles from Sam's. How fast do they drive?

– Contributed

40.

The numbers one through seven are drawn from a hat without replacement. In how many ways will the sequence be alternately increasing and decreasing after the first choice, assuming a random selection? *– Contributed*

41.



Three farmers, Adams, Brown and Clark all have farms containing the same number of acres. Adams' farm is most nearly square, the length being only 8 miles longer than the width. Clark has the most oblong farm, the length being 34 miles longer than the width. Brown's farm is intermediate between these two, the length being 28 miles longer than the width. If all the dimensions are in exact miles, what is the size of each farm? — *Contributed*

ANSWERS

1. He arrives at the North Pole having traversed a distance of $\sqrt{2} \cdot 10^7$ meters.

2. Very easily. He takes the center square and then counters each of his opponent's moves by taking the diametrically opposite square.

3. A triangle can be formed if and only if: (1) the breaks are on opposite sides of the midpoint of the Alpenstock (probability ½) and (2) the breaks are less than ½ an Alpenstock's distance apart (probability ½). The desired probability is therefore ¼.

← ASSUMING (1).

4. Seven. Two little girls and a boy, their father and mother and their father's father and mother.

5. $998{,}001 = (998 + 001)^2$ and $494{,}209 = (494 + 209)^2$.

6. My house number is 204, and there are 288 houses in my road.

7. The dimensions of the sails are: (1) $x = 6$, $y = 8$, $z = 10$; area, 24 sq. yds. and (2) $x = 12$, $y = 5$, $z = 13$; area, 30 sq. yds.

8. Dr. LaRouche was buying numbers (for doors, gates, etc.) and the price was 10 cents per digit.

9. Exactly 600 yards. It can be shown that in any general triangle the shortest line dividing it in two equal areas is equal to $\sqrt{2(s-a)(s-b)}$, where $s =$ semi-perimeter, and a and b are the two longer sides. The fence will terminate at the two longer sides.

10. $1001! + 2$; $1001! + 3, \ldots\ldots, 1001! + 1001$. $n! + A$ is divisible by A as long as $A > 1$ and $< (n + 1)$.

11. Since there is only one winner there must be n losers requiring n matches.

12. 153846

13. No. Removing opposite corners leaves 32 squares of one color and 30 squares of another color. However, each domino covers one square of each color.

14. The ladder will touch the wall 3.62 or 1.38 feet from the floor.

15. The missing number is 31. The terms of the sequence are the representations of 16 to various bases starting with base 16 and proceeding consecutively to base 2.

16. The distance from Kroflite to Beeline must be at least 25 miles. The towns could then be located at distances 0,1,4, 10,18,23 and 25 miles from Kroflite. There are 21 distances between towns and these are all distinct. Any shorter distance would mean at least one duplication.

17. $N = (9 + 2)\ 4!\ 5! = (11)\ (24)\ (120) = 31{,}680$.

18. The river flowed at 3 miles per hour.

19. There are 80 steps in the escalator.

20. Let A and B be the extremities of the diameter. Construct the line through P and A intersecting the circle at C, and let D be the corresponding point on the circle by considering the line through P and B. The intersection of the lines through A and D, B and C, yields a point E with the line through P and E perpendicular to the line through A and B.

21. In order for two similar triangles to have two sides the same, the sides must be of the form pm^3, $pm^2 n$ and $pm n^2$ for the first triangle and $pm^2 n$, $pm n^2$ and pn^3 for the corresponding sides of the second. The difference between the two sides not common to the two triangles is, therefore, $p (m^3 - n^3)$. The only solution of $p (m^3 - n^3) = 387$ in positive integers is $p = 1$, $m = 8$, $n = 5$. The triangles, therefore, have sides of 512, 320, 200, and 320, 200, 125.

22. $N = 120$.

23. A little time will reveal KING = 3721 and KNIGHT = 327184.

24. The point values must be coprime; otherwise there are infinitely many impossible scores. The number of impossible scores is then: $\frac{1}{2}(x-1)(y-1) = 35$. Hence x and y are (2,71), (3,36), (6,15), or (8,11). (3,36) and (6,15) are eliminated by the coprimality condition, and since a score of 58 could be attained by making 29 two point free throws, the value of a free throw is 8 and that of a field goal is 11.

25. Use of log table or slide rule involves unnecessary inaccuracy and time consumption. Simply raise both numbers to the 30th power, giving 1000 and 1024 respectively. Hence the cube root of two is larger.

26. $x^3 + y^3 = 12(x + y)$ or $x^2 - xy + y^2 = 12$, $x^2 - 2xy + y^2 = 12 - xy$, $(x - y)^2 = 12 - xy \geqq 0$. If $x \leqq y$ then $12 \geqq x^2$ and $x = 1, 2, 3$, The only integral solution is $x = 2$, $y = 4$.

27. The smallest integral solution satisfying the given conditions is $a = 25$, $b = 16$, $c = 39$.

28. Starting from his home country the traveler has three possible choices. After visiting the second country he has two choices, and after visiting the third country, two more choices. But after the first four countries have been visited, the second half of his trip is uniquely determined. There are, therefore, 3x2x2 = 12 possible ways of making the trip.

29. 1, 2, 3. $x + y + z = xyz$. So $\frac{1}{yz} + \frac{1}{xz} + \frac{1}{xy} = 1$. If $x \leqq y \leqq z$ then $\frac{1}{xy} \geqq \frac{1}{yz}$ and $\frac{1}{xz}$, so $\frac{3}{xy} \geqq 1$ and $xy \leqq 3$.

30. 16 x 30 square yards; diagonal = 34 yards, total fencing = 126 yards.

31. Each is making $\frac{11}{10} \cdot \frac{9}{10} = \frac{99}{100}$ times the original salary.

32. Any number of the form abcabc will do, since abcabc/abc equals 1001 which equals $7 \cdot 11 \cdot 13$.

33. R G R B R G R. He now cannot use R for then there would be two consecutive R's. He cannot use G because there would be two consecutive R G's. He cannot use B for then there would be two consecutive R G R B's. Any other pattern for the first seven stones would have allowed a choice for the eighth not involving a repetition.

34. N if we consider the letters as the first letters in one, two, three, four, five, six, seven, eight, nine.

35. $p = 2$, $q = 2$.

36. 480 husbands are taller and heavier than their wives.

37. If n is odd, 14^n ends in 4. So does 11×14^n. Therefore $(11 \times 14^n) + 1$ ends in 5 and hence is divisible by 5. If n is even, say equal to 2 m, $(11 \times 14^n) + 1$ is equal to $(11 \times 196^m) + 1$. Since $196 = 3 \times 65 + 1$, $(3 \times 65 + 1)^m$ will always leave a remainder of 1 when divided by 3. Therefore $(11 \times 196^m) + 1$ is equal to $11\,(3k + 1) + 1$ or $33\,k + 12$. Hence when n is even, $(11 \times 14^n) + 1$ is always divisible by 3. Therefore $(11 \times 14^n) + 1$ is never a prime, being a multiple of 5 or 3 according as n is odd or even.

38. $R = 18$, Area $= 1296\pi$, Volume $= 7776\pi$.

39. Furbisher drives 72 mph, LaRouche drives 36 mph.

40. 272.

41. If we let the widths of the three farms be A, B, and C, we have $A\,(A + 8) = B\,(B + 28) = C\,(C + 34)$. The equation $A\,(A + 8) = B\,(B + 28)$ has solutions $A = 8$, $B = 4$ and $A = 40$, $B = 32$. Only the latter satisfies the third equation, yielding the value $C = 30$. The farms are, therefore, 40x48, 32x60 and 30x64, all containing 1920 sq. mi.

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